Abstract

A systematic search for cycles in the generalized *3x+r* - problem exhibits some nice heuristical pattern, which I try to formalize here. The reader can find more formal and extensive consideration of this in Crandall's 1978 [Cra78], Lagarias' 1990 [Lag90] and Belaga's 2007 [Bel07] articles.

Preliminaries and notation

General transformation of one and of *N* steps

Let's denote one step of transformation from odd a_k to odd a_{k+1} in the Syracuse-style by

1.1) $a_{k+1} = (3a_k + r)/2^{A_k}$,

When iterated to N steps (writing S for the sum of all A_k) then the complete transformation can algebraically be expanded as

1.2) $a_{N+1} = a_1 \cdot 3^N / 2^S \quad + r \cdot (3^{N-1} + 3^{N-2} 2^{A_1} + 3^{N-3} 2^{A_1+A_2} + ... + 2^{A_1+...+A_{N-1}})/2^S$

Much interestingly, the parameter r can be brought outside of the parenthese. The parenthese itself should thus be seen as some "canonical" or "basic" composition over all the following discussion. Thus we introduce a short form $Q()$ for writing the parenthese expression by the vector of exponents only:

1. 3) $Q([A_1, A_2, ... , A_N]) = (3^{N-1} + 3^{N-2} 2^{A_1} + 3^{N-3} 2^{A_1+A_2} + ... + 2^{A_1+...+A_{N-1}})$

such that *1.2)* assumes the shorter form

1.4) $a_{N+1} = a_1 \cdot 3^N / 2^S \quad + r \cdot Q([A_1, A_2, ... , A_N]) / 2^S$

Let's give such vector an even shorter reference (although that shorter reference is formally underdetermined)

1.5) $E_{N,S} = [A_1, A_2, ... , A_N]$

such that in general we can write

1.6) $a_{N+1} = a_1 \cdot 3^N / 2^S \quad + r \cdot Q(E_{N,S}) / 2^S$ *for the general N-fold iterated transformation*

Let's give such transformation a symbolic name using letter ***T*** with suffix for the parameter r:

1.7) $a_{N+1} = T_r (a_1; [A_1, A_2, ... , A_N])$
$\quad = T_r(a_1 ; E_{N,S})$

Cycling

If we assume that this transformation defines a cycle for some a_1, meaning that $a_{N+1} = a_1$, then we can reformulate

1.8) $a_1 = T_r (a_1; E_{N,S})$
$a_1 = a_1 \cdot 3^N / 2^S \quad + r \cdot Q(E_{N,S}) / 2^S$ *for the definition of a cycle by* $\mathbf{E_{N,S}}$ *which occurs if* a_1 *is positive integer*
$a_1 \cdot 2^S = a_1 \cdot 3^N \quad + r \cdot Q(E_{N,S})$
$a_1 (2^S - 3^N) = r \cdot Q(E_{N,S})$

1.8.a) $a_1 = r \cdot Q(E_{N,S}) / (2^S - 3^N)$ *for the computation of the first element* a_1 *of a cycle given the exponents* A_k

For the classic Collatz-problem the parameter is $r=1$ and we know, that all $a_1 < 2^{60}$ converge to 1 so the cycle at $a_1=1$ is ***likely*** the only one.

The Collatz-conjecture for the case $r=1$ assumes that ***all*** positive integers a_1 fall down to the "trivial cycle" *1->1*->... when iteratively transformed by $T_1()$. That conjecture can be separated into two partial conjectures:

the *no-divergence-conjecture* meaning that there exist no divergent trajectories.

the *(no-nontrivial-) cycle*-conjecture meaning that there are no cycles besides the trivial one
(sometimes called "weak Collatz conjecture")

In the following ***we assume the truth*** of the Collatz-cycle-conjecture - such that the ***trivial cycle*** at $a_1=1$ and $A_1=2$ ***is the only one***.

A formula for the "trivial cycle" is from definition *(1.1)*

1.9) $a_1 = (3a_1+1)/2^A$

solving

$a_1 2^A - 3a_1 = 1$
$a_1 = (1)/(2^A - 3)$
allows the only possible solution having A=2 to get the positive integer $a_1=1$ *(A=1 would lead to* $a_1=-1$ *instead)*
$1 = (1)/(2^2 - 3)$

or, including explicitely the term for r=1 and the Q()-notation with N=1 and S=A=2,

$1 = 1 \cdot Q([2]) / (2^2 - 3^1) \quad = 1 \cdot Q([2,2]) / (2^4 - 3^2) \quad = 1 \cdot Q([2,2,2]) / (2^6 - 3^3) \quad = ...$
$= 1 \cdot (3^0)/(2^2 - 3^1)$
$= 1/1$

The assumption of the truth of the Collatz-cycle-conjecture can be reformulated such that in *(1.8a)* with $r=1$ no other positive integer solution for a_1 can be found, irrespectively of the number N and of the values of the exponents A_k:

Collatz-cycles conjecture: *in (8.d) the only solution for* $a_1>0$ *and r=1 and any N for*

1.10) $a_1 = 1 \cdot Q([A_1, A_2, A_3, ..., A_N]) / (2^S - 3^N)$
is
$a_1=1$ *and* $A_1=A_2=...=A_N=2$ *(and thus* $S=2 \cdot N$*)*

Insight in the generalized transformation 3x+r concerning cycles

The generalized trivial cycles

The first insight into the generalization with arbitrary odd parameter r is that from eq *(1.8a)* with $N=1$ and $S=2$

$$a_1 = r \cdot (3^0)/(2^2-3^1) \qquad [= r \cdot 1]$$

it follows immediately that we have a trivial-cycle-solution at $a_1=r$:

$$2.1) \qquad r\cdot 1 = r\cdot (3^0)/(2^2-3^1)$$

$$\implies \quad a_1 = r \quad \text{allows the generalized "trivial" cycle for any } r$$

The generalized non-trivial cycles

The next insight: for the non-trivial case we start at our form *(1.8a)*

$$2.2) \qquad a_1 = r \cdot Q(E_{N,S})/(2^S - 3^N)$$
$$= r \cdot Q([A_1, A_2, ... ,A_N])/(2^S - 3^N)$$

We denote the cancelled form of $Q(E_{N,S})/(2^S - 3^N)$ by the rational number "p/q" *writing:*

$$2.2a) \qquad a_1 = r \cdot (p/q)$$

Although (we assume that) in the Collatz-cycle-problem with $r=1$ there is no nontrivial solution for this, it is obvious that we need only that r is some multiple of q to have a solution for this cycle equation in integer a_1. Let r be any multiple of q, say $r=tq$, then

$$2.2b) \qquad a_1 = tq \cdot (p/q)$$
$$a_1 = tp \qquad \implies \quad a_1 = tp \text{ is a member of a cycle in } T_r(E_{N,S}) \text{ with parameter } r=tq$$

So for instance for $2^5-3^3=5$ we'll have a generalized nontrivial trivial cycle for $r=5$. For $2^8-3^5=13$ we'll have one with $r=13$, and so on.

Example: a short list for some small parameters r.

We begin with one more detailed description for $r=5$.

Heuristically (testing a_1 up to *1 000 000*) a table of results shows *6* cycles.

$r=5$

```
amin  relfreq%     N    S   vector          // comments
-----------------------------------------------------------------------------
   5 20.000000     1    2  [2]              // r·(2^S - 3^N) = r·1      generalized "trivial" cycle
    ------------------------------------------------------------------------
   1 14.154000     1    3  [3]              // 2^S - 3^N = 2^3 - 3^1 = 5 = r
   -    -    -    -    -    -    -    -    -    -    -    -    -    -    -    -    -    -
  19 49.522000     3    5  [1, 1, 3]        // 2^S - 3^N = 2^5 - 3^3 = 5 = r
  23  9.298000     3    5  [1, 2, 2]

 187  3.254000    17   27  [1, 1, 1, 1, 1, 2, 1, 1, 2, 1, 2, 3, 2, 1, 1, 1, 5]
 347  3.772000    17   27  [1, 1, 1, 1, 3, 1, 1, 1, 1, 2, 2, 4, 1, 2, 1, 2, 2]
```

Legend: the first column indicates the minimal element of the described cycle; for instance, in the same way as in the $3x+1$-problem we have the trivial cycle at $a_{min}=1$, we have in the $3x+5$-problem the trivial cycle at $a_{min}=5$. That cycle has of course the length $N=1$ and vector of exponents is *[2]* , so also the sum S of exponents is $S=2$. (Even more, in general, having a cycle at a_1, N can of course be any number, then $\mathbf{E}_{N,S} =[2,2,2,2,...,2]$ with N elements of value *2* and $S=2N$ - this all defines the same trivial cycle)

The second column gives a statistic *"rel freq%"* in percent: for how many a_1 in that tested range occurs that specific cycle (relatively to size of range, in percent).

The other columns have obvious descriptions in their title.

First we find the generalized "trivial cycle" at $a_1=5$.

A second cycle of length $N=1$ occurs at $a_{min}=a_1=1$ because we have $(1\cdot 3+5)/2^3 = 1$ and the vector of exponents is simply *[3]* and also $S=3$.

A third cycle occurs with length $N=3$; using the exponents *[1,1,3]* (and thus $S=5$ - we want that $2^S > 3^N$) it can be determined by

$$Q([1,1,3])=3^2 + 3\cdot 2^1+2^{1+1} = 19$$
$$2^S - 3^N = 5$$
$$a_1 = r \cdot Q([1,1,3])/(2^S-3^N) = 5 \cdot 19/5 = 19$$

and a fourth one by

$$Q([1,2,2])=3^2 + 3\cdot 2^1+2^{1+2} = 23$$
$$2^S - 3^N = 5$$
$$a_1 = r \cdot Q([1,2,2])/(2^S-3^N) = 5 \cdot 23/5 = 23$$

We have even two longer cycles with $N=17$, $S=27$, and $a_1=187$ and $a_1=347$. Here the expression 2^S-3^N has the following factorization:

$$factors(2^{27} - 3^{17}) = 5 \cdot 71 \cdot 14303$$

The p/q-notation says

$$a_1 = r \cdot Q(E_{17,27}) / 5 \cdot 71 \cdot 14303 = r \cdot p/q$$

and obviously the two expressions for $Q(E_{17,27})$ cancel the factors $71\cdot 14303$ and we need $r=5$ to find an integer value for a_1 and establish a nontrivial cycle for this parameter $r=5$.

Here is a compacted table for some small problem parameters *r*: in parentheses *(N,S)* and the number(s) after the parentheses is/are a_1. If there are more numbers here, then this shows occurence of more cycles with the same *(N,S)* (true new cycle, no rotations!)

Table 2:

r	"trivial"	non-trivial	cycles				comments
1	(1,2):1						
3	(1,2):3						
5	(1,2):5	(1,3):1	(3,5):19,23	(17,27):187,347			
7	(1,2):7				(2,4):5		
9	(1,2):9						
11	(1,2):11	(2,6):1	(8,14):13				
13	(1,2):13	(1,4):1	(5,8):211,259, 227, 287,251, 283,319	(15,24):131			
15	(1,2):15	(1,3):3	(3,5):57,69	(17,27):561,1041			*The yellow marked cycles are inherited from that of r=5 because of the factorization of r=5·3*
17	(1,2):17	(2,7):1		(18,31):23			
19	(1,2):19		(5,11):5				
21	(1,2):21				(2,4):15		
23	(1,2):23		(2,5):5,7	(26,43):41			
25	(1,2):25	(1,3):5	(3,5):95,115	(17,27):935, 1735	(8,16):7	(4,8):17	*The yellow marked cycles are inherited from that of r=5 because of the factorization of r=5·5*
27	(1,2):27						
29	(1,2):29	(1,5):1	(9,17):11	(41,65):3811,7055			
31	(1,2):31		(12,23):13				
33	(1,2):33	(2,6):3	(8,14):39				
35	(1,2):35	(1,3):7	(3,5):133,161	(17,27):1309,2429	(2,4):25	(4,8):13,17	*The yellow marked cycles are inherited from that of r=5 because of the factorization of r=5·7 and the green marked cycle is inherited from that of r=7 . The new cycle (4,8) is specific for r*
.....	...	...	...	...	...	...	

See a longer table in Belaga/Mignotte: Cyclic structure of Dynamical Systems (2000)
(*http://hal.archives-ouvertes.fr/IRMA-ACF*, file *hal-00129656* according to Lagarias (2011))

Parameter $r=(2^S-3^N)$ with given N and S

In an equation with given *N* and *S* where also $r=(2^S - 3^N)$ and thus

2.3) $a_1 = r \cdot Q(E_{N,S}) / (2^S - 3^N)$
$a_1 = Q(E_{N,S})$

we have obviously that any vector $E_{N,S}$ gives one valid solution for a cycle containing a_1. So any vector

$$E_{N,S} = [A_1, A_2, A_3, \dots, A_N] \qquad \text{restricted to} \qquad \sum_{k=1}^{N} A_k = S$$

we have a combinatorical number C^* of members $a_{c,k}$ of cycles, and since *N* members define one cycle we have $C=C^*/N$ cycles for this parameter *r*.

Example 1: let *N=3*, *S=5*. Then let $r=2^S-3^N= 32 - 27 = 5$. Then the list of possible vectors $E_{N,S}$ is

[1,1,3], [1,2,2], [2,1,2], [1,3,1], [2,2,1], [3,1,1]

Thus $C^*=6$, $C=6/3=2$ and we shall have *2* different cycles with this *r=5*. Note that the vectors above can be seen as rotations, such that we get the two groups

[3,1,1], [1,3,1], [1,1,3]	*cycle 1*	*(49 -> 31 -> 19 -> ...)*
[2,2,1], [2,1,2], [1,2,2]	*cycle 2*	*(37 ->29 -> 23 -> ...)*

which define "rotationally" the according members a_1, a_2, a_3 for each cycle when formula *2.3)* is applied accordingly.

Example 2: let *N=5*, *S=8*. Then let $r=2^8-3^5= 256 - 243 = 13$. Then the list of possible vectors $E_{N,S}$ is

[4, 1, 1, 1, 1] [3, 2, 1, 1, 1] [2, 3, 1, 1, 1] [1, 4, 1, 1, 1] [3, 1, 2, 1, 1]
[2, 2, 2, 1, 1] [1, 3, 2, 1, 1] [2, 1, 3, 1, 1] [1, 2, 3, 1, 1] [1, 1, 4, 1, 1]
[3, 1, 1, 2, 1] [2, 2, 1, 2, 1] [1, 3, 1, 2, 1] [2, 1, 2, 2, 1] [1, 2, 2, 2, 1]
[1, 1, 3, 2, 1] [2, 1, 1, 3, 1] [1, 2, 1, 3, 1] [1, 1, 2, 3, 1] [1, 1, 1, 4, 1]
[3, 1, 1, 1, 2] [2, 2, 1, 1, 2] [1, 3, 1, 1, 2] [2, 1, 2, 1, 2] [1, 2, 2, 1, 2]
[1, 1, 3, 1, 2] [2, 1, 1, 2, 2] [1, 2, 1, 2, 2] [1, 1, 2, 2, 2] [1, 1, 1, 3, 2]
[2, 1, 1, 1, 3] [1, 2, 1, 1, 3] [1, 1, 2, 1, 3] [1, 1, 1, 2, 3] [1, 1, 1, 1, 4]

Thus $C^*=35$, $C=35/5=7$ and we shall have *7* different cycles with this *r=13*.

Again the vectors above can be separated into groups of rotations, such that we get the *7* groups

[4, 1, 1, 1, 1] [1, 4, 1, 1, 1] [1, 1, 4, 1, 1] [1, 1, 1, 4, 1] [1, 1, 1, 1, 4]	*cycle 1*	*211 -> 323 -> 491 -> 743 -> 1121 -> 211*
[3, 2, 1, 1, 1] [1, 3, 2, 1, 1] [1, 1, 3, 2, 1] [1, 1, 1, 3, 2] [2, 1, 1, 1, 3]	*cycle 2*	*259 -> 395 -> 599 -> 905 -> 341 -> 259*
[2, 3, 1, 1, 1] [1, 2, 3, 1, 1] [1, 1, 2, 3, 1] [1, 1, 1, 2, 3] [3, 1, 1, 1, 2]	*cycle 3*	*227 -> 347 -> 527 -> 797 -> 601 -> 227*
[3, 1, 2, 1, 1] [1, 3, 1, 2, 1] [1, 1, 3, 1, 2] [2, 1, 1, 3, 1] [1, 2, 1, 1, 3]	*cycle 4*	*287 -> 437 -> 331 -> 503 -> 761 -> 287*
[2, 1, 3, 1, 1] [1, 2, 1, 3, 1] [1, 1, 2, 1, 3] [3, 1, 1, 2, 1] [1, 3, 1, 1, 2]	*cycle 5*	*251 -> 383 -> 581 -> 439 -> 665 -> 251*
[2, 2, 2, 1, 1] [1, 2, 2, 2, 1] [1, 1, 2, 2, 2] [2, 1, 1, 2, 2] [2, 2, 1, 1, 2]	*cycle 6*	*283 -> 431 -> 653 -> 493 -> 373 -> 283*
[2, 2, 1, 2, 1] [1, 2, 2, 1, 2] [2, 1, 2, 2, 1] [1, 2, 1, 2, 2] [2, 1, 2, 1, 2]	*cycle 7*	*319 -> 485 -> 367 -> 557 -> 421 -> 319*

It comes out, that the number C^* is simply determined by $C^*=binomial(S-1,N-1)$.

Unfortunately, this is not always divisible by *N* so the $C=C^*/N$ rule cannot always be applied.

Parameter r with $(2^S-3^N)=t \cdot s$ and $Q(E_{N,S})=t \cdot u$ (with given N and S)

If we have $(2^S-3^N)=t \cdot s$ and $Q(E_{N,S})=t \cdot u$ (with given *N* and *S*) , then *r* needs to cancel the factor *s* in the denominator. If *r=s* then we have a cycle with $a_1=u$ and if *r* is a *j*'th multiple of *s* we have a cycle with $a_1=j \cdot u$

Example: *N=3, S=21* then $(2^S-3^N)=t \cdot s = 19 \cdot (125 \cdot 883)$ and $Q([1,1,19])=t \cdot u=19 \cdot 1$. Let $r=125 \cdot 883= 110375 \cdot$

This gives $C^*=(21-1:5-1)=190$ including one *1-step* cycle, so the number $C=(C^*-1)/N = 63$ gives the cycle candidates. But only some have the denominator's factor *t=19* in the primefactorization of the numerator $Q(E_{N,S})$ namely

	E_1	E_2	E_3	$Q(E_1)$	$Q(E_2)$	$Q(E_3)$	a_1 ->	a_2 ->	a_3 ->	comment
cycle 1	[7,7,7]			19·883			883	883	883	1-step-cycle
cycle 2	[1,1,19]	[1,19,1]	[19,1,1]	19	19·229·241	19·281·491	1	55189	137971	
cycle 3	[2,8,11]	[8,11,2]	[11,2,8]	19·5·11	19·5·5527	19·5·151	55	27635	755	
cycle 4	[3,13,5]	[13,5,3]	[5,3,13]	19·7·17·29	19·15091	19·19	3451	15091	19	

Of course, if it happens, that $r=110375$ can also expressed by a formula 2^S-3^N with some N and S then we have a further number $C^*=binomial(S-1,N-1)$ and an according set of cycles with cardinality $C=C^*/N$.

Composite parameter r vs. prime r

If we compare composite and prime r then we find one more interesting aspect (which was indicated already in the above table).

It appears that if r is composite with primefactors s and t, such that $r=s\cdot t$ then $T_r(\,)$ has the cycles of both $T_s(\,)$ and $T_t(\,)$ (only that the members a_k are rescaled by the according factor: $T_{st}(\,)$ has the same cycles as $T_s(\,)$ but the members are scaled by $r/s=t$ and as well the same as $T_t(\,)$ but those are scaled by $r/t = s$) - but moreover it seems, that generally $T_{st}()$ has some additional "*primitive*" cycles (except for additional powers of 3 in s or t).

So for instance, let $s=5, t=7, r=st=35$.

$T_5(\,)$ has the nontrivial cycles

$(N,S)=(1,3)$ $a_1=1$, $(N,S)=(3,5)$ $a_1=19$ and 23 , $(N,S)=(17,27)$ $a_1=187$ and 347

Then $T_{35}(\,)$ has (at least) the same cycles, but the members a_k are multiplied by $35/5=7$:

$(N,S)=(1,3)$ $a_1=7$, $(N,S)=(3,5)$ $a_1=133$ and 161 , $(N,S)=(17,27)$ $a_1=1309$ and 2429

$T_7(\,)$ has the nontrivial cycles

$(N,S)=(2,4)$ $a_1=5$

Then $T_{35}(\,)$ has (at least) the same cycles, but the members a_k are multiplied by $35/7=5$:

$(N,S)=(2,4)$ $a_1=25$

But additionally, each composite r may provide cycles which are *"unexplained"* by its primefactors.
*(In Lagarias [Lag90] and later Belaga [Bel07] this is called "**primitive cycle**" which I'll adapt here)*

So $T_{35}(\,)$ can have some additional, *primitive* cycles - when in the expression $a_1 = r\cdot Q(E_{N,S})/(2^S-3^N) = p/q$ the reduced denominator q carries both factors $s\cdot t$. In the current case we find

$(N,S)=(4,8)$ $a_1=13$ $(E_{4,8}=[1,1,1,5])$
and $a_1=17$ $(E_{4,8}=[1,2,1,4])$

This feature of additional *primitive* cycles is illustrated in the following table (in a hopefully more enlightening way). The leading column shows the problem parameters r. The heading lines give the arguments (N,S) and in the cells are the list of a_1, each defining a cycle. The cycles with $(1,2)$ are the trivial ones, the rescaled trivial cycle from the $3x+1$-problem.

Table 2 illustrating the occurences of cycles for $T_r(\,)$ with composite $r=stu=385=5\cdot 7\cdot 11$ compared with that for $T_r(\,)$ of prime $r=s$ $r=t$ $r=u$ $r=st$ $r=tu$ $r=su$. Entries are the minimal members a_1; if more than one cycle/minimal member a_1 exist they are separated by ";"

(N,S) / r	(1,2)	(1,3)	(3,5)	(17,27)	(2,4)	(4,8)	(2,6)	(8,14)	(4,12)	(16,28)	(16,38)	(6,18)	(20,40)	(48,84)
5	**5**	**1**	**19;23**	**187;347**										
35	35	7 *7=1·7*	133;161 *133=19·7* *161=23·7*	1309;2429 *1309=187·7* *2429=347·7*	25 *25=5·5*	**13;17** ***-primitive-***								
7	**7**				**5**									
77	77				55 *55=5·11*		7 *7=1·7*	91 *91=13·7*			**1** ***-primitive-***			
11	**11**						**1**	**13**						
55	55	11 *11=1·11*	209;253 *209=19·11* *253=23·11*	2057;3817 *2057=187·11* *3817=347·11*			5;7 *5=1·5* *7=1·7*	65 *65=13·5*	**1** ***-primitive-***	**41** ***-primitive-***				
385	385	77 *77=7·11*	1463;1771 *1463=19·11·7* *1771=23·11·7*	14399;26719	275 *275=5·11·5*	143;187 *143=13·11* *187=17·11*	35;49 *35=5·7* *49=7·7*	455 *455=13·7·5*	7 *7=1·7*	287 *287=41·7*	5 *5=1·5*	**17** ***-primitive-***	**23** ***-primitive-***	**107** ***-primitive-***

Additional remark: if we expand this table to contain also $T_{3\cdot 385}()$ then this does not add new cycles, because any factor of 3 in r (such that $r=3r'$) does not compensate for more factors in the denominator of p/q : in this type of problems (using $3a_1+r$) the denominator q cannot contain the factor 3 because q is a factor of 2^S-3^N which itself cannot contain that factor 3 except if $N=0$

Further discussion / draft material.

a) Discussing transformations and cycles in terms of exponents-vectors $E_{N,S} = [A_1, A_2, \dots, A_N]$ and stating equations *6)* give some interesting consideration:

- of course, ***any*** combination of values $A_k > 0$ in $E_{N,S}$ lead to a rational number
  $p/q = Q([A_1, A_2, \dots, A_N])/(2^S - 3^N)$
  (where *p/q means the fraction in most cancelled form)*
- thus ***every*** expression for the existence of a cycle $a_1 = r \cdot p/q$ can obviously be solved in a pair of integers $a_1=p, r=q$ which immediately means that
- ***for any exponents-vector*** $E_{N,S}=[A_1, A_2, \dots, A_N]$ a solution for ***a cycle exists*** with an appropriate parameter r .
  Moreover, ***multiple*** solutions exists by scaling of a_1 and r by some common cofactor.
- The truth of the Collatz-conjecture is then equivalent to the statement, that no exponents-vector $E_{N,S} = [A_1, A_2, \dots, A_N]$ exists such that $2^S > 3^N$ and $Q(E_{N,S})$ is divisible by 2^S-3^N except if all $A_k=2$ and thus $S=2N$. (For $2^S < 3^N$ and thus negative a_1 we know three more cycles)

b) and c) not yet worked out, see file Collatz_3x_r_material.doc .

A side remark: Analogy to the <u>Zsigmondy-theorem</u> on <u>Mersenne-numbers</u>: This seems to occur in a similar manner as it occurs with the prime factorization of the *Mersenne numbers* M_n : the M_n of composite indexes $n=pq$ have all primefactors as M_p and M_q alone, but always some additional primefactors (exept for $M_{2\cdot3}=M_6=3^2\cdot7$), which are called "*primitive primefactors*" in that context. This is a theorem known by a proof of ***Zsigmondy*** (*wikipedia*). I did not yet see the possibility to translate this to the problem here, however, and to do something like a proof. The striking similarity is on the exceptional cases: if r is composed with primefactor *3* then r has not those *"unexplained"* additional cycles (like M_6 has no "*unexplained*"/"primitive" primefactors)!

Appendix: List of cycles in *3x+r*-problem with odd parameters *r* with *5<=r <=63* and some more small specific cases

For cycle-searching I used odd numbers $a_k = 1 \dots 999\,999$.

The column "relfreq" gives the relative frequency (in percents) of the occurence of the cycle as the tail of the trajectories for the odd a_k in the interval $1 <= a_k <= 999\,999$. Occasionally I used some shorter computations $a_k = 1 \dots 19\,999$ only, because the relative frequencies appeared to be very stable and computation is time-consuming

r=1 (The original Collatz-problem; $1=2^2 - 3\cdot 1$: this leads systematically to a N=1,S=2 cycle at a_1=1).

```
  amin relfreq%     N    S   vector
------------------------------------------------------------------------------
     1 100.000      1    2  [2]                        The basic "trivial cycle" in the Collatzproblem
    --------------------------------------------------------------------------
   - 1              1    1  [1]                        There are cycles in the negative numbers but they are not considered furtherly here
   - 5              2    3  [1, 2]
   -17              7   11  [1, 1, 1, 2, 1, 1, 4]
```

r=3

```
  amin relfreq%      N    S   vector
------------------------------------------------------------------------------
     3 100.000      1    2  [2]
    --------------------------------------------------------------------------
```

r=5 ($5=2^3 - 3\cdot 1$: this leads systematically to a N=1,S=3 cycle at a_1=1)

```
  amin  relfreq%      N    S   vector         // comments
------------------------------------------------------------------------------
     5 20.000000    1    2  [2]             // r·(2^S - 3^N) = r·1
    --------------------------------------------------------------------------
     1 14.154000    1    3  [3]             // 2^S - 3^N = 5 = r
    -   -   -   -   -   -   -   -   -   -   -   -   -   -   -   -   -   -   -
    19 49.522000    3    5  [1, 1, 3]       // 2^S - 3^N = 5 = r
    23  9.298000    3    5  [1, 2, 2]

   187  3.254000   17   27  [1, 1, 1, 1, 1, 2, 1, 1, 2, 1, 2, 3, 2, 1, 1, 1, 5]
   347  3.772000   17   27  [1, 1, 1, 1, 3, 1, 1, 1, 1, 2, 2, 4, 1, 2, 1, 2, 2]
```

r=7

```
  amin  relfreq%      N    S   vector
------------------------------------------------------------------------------
     7 14.280000    1    2  [2]             // r·(2^S - 3^N) = r·1
    --------------------------------------------------------------------------
     5 85.720000    2    4  [1, 3]         // 2^4 - 3^2 = 7 = r
```

r=9

```
  amin relfreq%      N    S   vector
------------------------------------------------------------------------------
     9 100.000      1    2  [2]
    --------------------------------------------------------------------------
```

r=11

```
  amin relfreq%     N    S   vector
------------------------------------------------------------------------------
    11  9.080000    1    2  [2]
    --------------------------------------------------------------------------
     1 20.100000    2    6  [1, 5]          //    2^S - 3^2 = 5·r
    13 70.820000    8   14  [1, 1, 2, 2, 1, 1, 3, 3]
   -19              3    4  [1, 1, 2]
```

r=13 ($13=2^4 - 3\cdot 1$ this leads systematically to a N=1,S=4 cycle at a_1=1)

```
  amin  relfreq%      N    S   vector
------------------------------------------------------------------------------
    13  7.692000    1    2  [2]
    --------------------------------------------------------------------------
     1 47.550000    1    4  [4]               // 2^4 - 3^1 = 13 = r
    -   -   -   -   -   -   -   -   -   -   -   -   -   -   -   -   -   -   -
   211  3.334000    5    8  [1, 1, 1, 1, 4]   // 2^8 - 3^5 = 13 = r
   227  1.880000    5    8  [1, 1, 1, 2, 3]
   251  1.958000    5    8  [1, 1, 2, 1, 3]
   259  3.934000    5    8  [1, 1, 1, 3, 2]
   283  2.506000    5    8  [1, 1, 2, 2, 2]
   287  4.380000    5    8  [1, 2, 1, 1, 3]
   319  1.424000    5    8  [1, 2, 1, 2, 2]

   131 25.342000   15   24  [1, 1, 1, 3, 1, 1, 1, 1, 1, 2, 1, 2, 1, 2, 5]  // Q(EN,S)=131*186793   2^24-3^15 = 13*186793
```

r=15

```
  amin  relfreq%      N    S   vector
------------------------------------------------------------------------------
    15 20.000000    1    2  [2]
    --------------------------------------------------------------------------
     3 14.168000    1    3  [3]
    -   -   -   -   -   -   -   -   -   -   -   -   -   -   -   -   -   -   -
    57 49.642000    3    5  [1, 1, 3]
    69  9.336000    3    5  [1, 2, 2]
   561  3.176000   17   27  [1, 1, 1, 1, 1, 2, 1, 1, 2, 1, 2, 3, 2, 1, 1, 1, 5]
  1041  3.678000   17   27  [1, 1, 1, 1, 3, 1, 1, 1, 1, 2, 2, 4, 1, 2, 1, 2, 2]
```

r=17

```
  amin  relfreq%     N    S   vector
-----------------------------------------------------------------------------
    17  5.880000    1    2  [2]
    -------------------------------------------------------------------------
     1 33.300000    2    7  [2, 5]
    23 60.820000   18   31  [1, 1, 2, 1, 2, 3, 1, 1, 1, 2, 2, 2, 1, 1, 4, 1, 3, 2]
   -65              4    6  [1, 1, 1, 3]                  -65=13*-5
```

r=19

```
  amin  relfreq%     N    S   vector
-----------------------------------------------------------------------------
    19  5.260000    1    2  [2]
    -------------------------------------------------------------------------
     5 94.740000    5   11  [1, 1, 2, 4, 3]
```

r=21

```
  amin relfreq%     N    S   vector
-----------------------------------------------------------------------------
    21 14.2800      1    2  [2]
    -------------------------------------------------------------------------
    15 85.7200      2    4  [1, 3]
```

r=23

```
  amin  relfreq%     N    S   vector
-----------------------------------------------------------------------------
    23  4.348000    1    2  [2]
    -------------------------------------------------------------------------
     5  6.298000    2    5  [1, 4]          // 2^5 - 3^2 = r
     7 41.528000    2    5  [2, 3]

    41 47.826000   26   43  [1, 1, 1, 1, 1, 2, 1, 1, 2, 2, 1, 1, 1, 4, 2, 2, 1, 1, 1, 2, 5, 1, 1, 3, 2, 2]
```

r=25

```
  amin relfreq%    N    S   vector
-----------------------------------------------------------------------------
    25  4.0000    1    2  [2]
    -------------------------------------------------------------------------
     5  2.8800    1    3  [3]
    95  9.5600    3    5  [1, 1, 3]
   115  1.9400    3    5  [1, 2, 2]
    17 37.4200    4    8  [2, 1, 2, 3]
     7 42.5800    8   16  [1, 1, 1, 1, 2, 5, 1, 4]
   935  0.7900   17   27  [1, 1, 1, 1, 1, 2, 1, 1, 2, 1, 2, 3, 2, 1, 1, 1, 5]
  1735  0.8300   17   27  [1, 1, 1, 1, 3, 1, 1, 1, 1, 2, 2, 4, 1, 2, 1, 2, 2]
```

r=27

```
  amin relfreq%     N    S   vector
-----------------------------------------------------------------------------
    27 100.000      1    2  [2]
    -------------------------------------------------------------------------
```

r=29 *(=2^5 - 3·1 this leads systematically to a N=1,S=5 cycle at a_1=1))*

```
  amin  relfreq%     N    S   vector
-----------------------------------------------------------------------------
    29  3.448000    1    2  [2]
    -------------------------------------------------------------------------
     1  7.922000    1    5  [5]               //  2^5 - 3^1 = r
    -   -   -   -   -   -   -   -   -   -   -   -   -   -   -   -   -   -   -
    11 87.984000    9   17  [1, 1, 2, 2, 1, 2, 1, 3, 4]

  3811  0.354000   41   65  [1, 1, 1, 1, 1, 1, 1, 2, 3, 1, 1, 1, 2, 1, 1, 2, 1, 1, 1, 1, 1, 1, 2, 1, 1, 1, 3, 1, 1, 5, 1, 2, 2, 1, 1, 1, 1, 6, 2, 3, 3]
  7055  0.292000   41   65  [1, 2, 1, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 2, 3, 1, 2, 2, 1, 1, 1, 1, 1, 1, 2, 1, 5, 3, 1, 3, 1, 1, 1, 1, 3, 2, 1, 2, 2, 3, 2]
```

Note that (41,65) are convergents of the continued fraction

r=31

```
  amin  relfreq%     N    S   vector
-----------------------------------------------------------------------------
    31  3.226000    1    2  [2]
    -------------------------------------------------------------------------
    13 96.774000   12   23  [1, 3, 1, 1, 1, 3, 1, 2, 2, 3, 1, 4]
```

r=33

```
  amin relfreq%    N    S   vector
-----------------------------------------------------------------------------
    33 9.09000    1    2  [2]
    -------------------------------------------------------------------------
     3 19.5600    2    6  [1, 5]
    39 71.3500    8   14  [1, 1, 2, 2, 1, 1, 3, 3]
```

r=35

```
  amin relfreq%    N    S   vector
-----------------------------------------------------------------------------
    35 2.85000    1    2  [2]
    -------------------------------------------------------------------------
     7 1.98000    1    3  [3]
```

```
   25 17.1500    2    4  [1, 3]

  133 6.91000    3    5  [1, 1, 3]
  161 1.42000    3    5  [1, 2, 2]
   13 17.2200    4    8  [1, 1, 1, 5]
   17 51.3500    4    8  [1, 2, 1, 4]

 1309 0.490000   17   27  [1, 1, 1, 1, 1, 2, 1, 1, 2, 1, 2, 3, 2, 1, 1, 1, 5]
 2429 0.630000   17   27  [1, 1, 1, 1, 3, 1, 1, 1, 1, 2, 2, 4, 1, 2, 1, 2, 2]
```

r=37

```
  amin relfreq%    N    S   vector
---------------------------------------------------------------------------
   37 2.71000    1    2  [2]
    -----------------------------------------------------------------------
   19 48.3500    3    6  [1, 1, 4]
   23 23.0900    3    6  [1, 2, 3]
   29 25.8500    3    6  [2, 1, 3]
```

r=39

```
  amin relfreq%    N    S   vector
---------------------------------------------------------------------------
   39 7.69000    1    2  [2]
    -----------------------------------------------------------------------
    3 47.5100    1    4  [4]

  633 3.12000    5    8  [1, 1, 1, 1, 4]
  777 3.92000    5    8  [1, 1, 1, 3, 2]
  681 1.79000    5    8  [1, 1, 1, 2, 3]
  861 4.57000    5    8  [1, 2, 1, 1, 3]
  753 2.05000    5    8  [1, 1, 2, 1, 3]
  849 2.66000    5    8  [1, 1, 2, 2, 2]
  957 1.36000    5    8  [1, 2, 1, 2, 2]

  393 25.3300   15   24  [1, 1, 1, 3, 1, 1, 1, 1, 1, 2, 1, 2, 1, 2, 5]
```

r=41

```
   amin relfreq%   N    S   vector
---------------------------------------------------------------------------
   41 2.44000    1    2  [2]
    -----------------------------------------------------------------------
    1 97.5600    8   20  [2, 1, 3, 1, 2, 1, 1, 9]
```

r=43

```
   amin relfreq%   N    S   vector
---------------------------------------------------------------------------
   43 2.32000    1    2  [2]
    -----------------------------------------------------------------------
    1 97.6800    3   11  [1, 4, 6]
```

r=45

```
   amin relfreq%   N    S   vector
---------------------------------------------------------------------------
   45 20.0000    1    2  [2]
    -----------------------------------------------------------------------
    9 14.4500    1    3  [3]
  171 48.4900    3    5  [1, 1, 3]
  207 9.57000    3    5  [1, 2, 2]
 1683 3.82000   17   27  [1, 1, 1, 1, 1, 2, 1, 1, 2, 1, 2, 3, 2, 1, 1, 1, 5]
 3123 3.67000   17   27  [1, 1, 1, 1, 3, 1, 1, 1, 1, 2, 2, 4, 1, 2, 1, 2, 2]
```

r=47 (=2^7-3^4)

```
   amin relfreq%   N    S   vector
---------------------------------------------------------------------------
   47 2.13000    1    2  [2]
    -----------------------------------------------------------------------
   65 13.5300    4    7  [1, 1, 1, 4]
   89 5.94000    4    7  [1, 1, 3, 2]
   73 5.76000    4    7  [1, 1, 2, 3]
   85 3.91000    4    7  [1, 2, 1, 3]
  101 3.28000    4    7  [1, 2, 2, 2]
    5 39.2900    7   18  [1, 2, 3, 3, 1, 4, 4]
   25 26.1600   16   28  [1, 1, 3, 1, 1, 1, 1, 2, 1, 2, 1, 3, 2, 1, 2, 5]
```

r=49

```
   amin relfreq%   N    S   vector
---------------------------------------------------------------------------
   49 2.04000    1    2  [2]
    -----------------------------------------------------------------------
   35 12.2400    2    4  [1, 3]
   25 85.7200   22   38  [2, 1, 1, 1, 3, 2, 1, 1, 1, 1, 1, 1, 2, 3, 1, 1, 2, 2, 1, 2, 2, 6]
  -65            4    5  [1, 1, 1, 2]
```

r=51

```
   amin relfreq%   N    S   vector
---------------------------------------------------------------------------
   51 5.88000    1    2  [2]
    -----------------------------------------------------------------------
    3 32.5200    2    7  [2, 5]
   69 61.6000   18   31  [1, 1, 2, 1, 2, 3, 1, 1, 1, 2, 2, 2, 1, 1, 4, 1, 3, 2]
```

r=53

```
   amin relfreq%   N    S   vector
-----------------------------------------------------------------------------
    53 1.89000    1    2  [2]
    -------------------------------------------------------------------------
   103 98.1100   17   29  [1, 2, 2, 2, 1, 1, 1, 1, 2, 2, 3, 1, 3, 1, 1, 2, 3]
```

r=55

```
   amin relfreq%   N    S   vector
-----------------------------------------------------------------------------
    55 1.82000    1    2  [2]
    -------------------------------------------------------------------------
    11 1.29000    1    3  [3]
     5 3.88000    2    6  [1, 5]
     7 38.9300    2    6  [2, 4]
   209 4.30000    3    5  [1, 1, 3]
   253 0.88000    3    5  [1, 2, 2]
     1 16.3000    4   12  [1, 1, 2, 8]
    65 14.3000    8   14  [1, 1, 2, 2, 1, 1, 3, 3]
    41 17.5000   16   28  [1, 1, 1, 1, 2, 1, 1, 3, 1, 2, 2, 4, 1, 2, 2, 3]
  2057 0.38000   17   27  [1, 1, 1, 1, 1, 2, 1, 1, 2, 1, 2, 3, 2, 1, 1, 1, 5]
  3817 0.42000   17   27  [1, 1, 1, 1, 3, 1, 1, 1, 1, 2, 2, 4, 1, 2, 1, 2, 2]
```

r=57

```
   amin relfreq%   N    S   vector
-----------------------------------------------------------------------------
    57 5.26000    1    2  [2]
    -------------------------------------------------------------------------
    15 94.7400    5   11  [1, 1, 2, 4, 3]
```

r=59

```
   amin relfreq%   N    S   vector
-----------------------------------------------------------------------------
    59 1.69000    1    2  [2]
    -------------------------------------------------------------------------
     1 23.3300   11   28  [1, 3, 2, 1, 1, 2, 5, 1, 1, 4, 7]
   133 53.9600    6   10  [1, 1, 1, 1, 1, 5]
   181 10.4700    6   10  [1, 1, 1, 3, 1, 3]
   185 3.44000    6   10  [1, 2, 1, 1, 1, 4]
   217 2.42000    6   10  [1, 2, 1, 2, 1, 3]
   149 2.37000    6   10  [1, 1, 1, 2, 1, 4]
   221 2.32000    6   10  [1, 1, 2, 2, 2, 2]
```

r=61 (=2^6-3·1 this leads systematically to a N=1,S=6 cycle at a1=1)

```
   amin relfreq%   N    S   vector
-----------------------------------------------------------------------------
    61 1.64000    1    2  [2]
    -------------------------------------------------------------------------
     1 93.3500    1    6  [6]
   235 5.01000   41   66  [1, 1, 2, 2, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 2, 1, 1, 1, 3, 1, 2, 2, 2, 3, 1, 1, 2, 3, 2, 1, 1, 1, 2, 3, 1, 2, 3, 2, 1, 3, 2]
```

r=63

```
   amin relfreq%   N    S   vector
-----------------------------------------------------------------------------
    63 14.2800    1    2  [2]
    -------------------------------------------------------------------------
    45 85.7200    2    4  [1, 3]
```

r=73

```
   amin relfreq%     N    S   vector
-----------------------------------------------------------------------------
    73  1.373333    1    2  [2]
    -------------------------------------------------------------------------
     5  9.500000    4   12  [3, 1, 3, 5]
    47 39.800000    8   15  [1, 1, 3, 1, 2, 1, 3, 3]
    19 49.326667   32   60  [1, 2, 1, 2, 2, 4, 2, 3, 1, 1, 1, 1, 5, 2, 2, 1, 1, 1, 1, 1, 1, 2, 2, 1, 3, 1, 1, 2, 1, 1, 2, 8]
```

r=77

```
   amin relfreq%      N    S   vector
-----------------------------------------------------------------------------
      77 1.30000     1     2  [2]
    -------------------------------------------------------------------------
      55 7.79000     2     4  [1, 3]
       7 2.78000     2     6  [1, 5]
      91 10.2000     8    14  [1, 1, 2, 2, 1, 1, 3, 3]
       1 77.9300    16    38  [4, 2, 1, 3, 2, 1, 2, 2, 1, 2, 1, 1, 2, 4, 1, 9]
```

r=125 (=2^7-3·1   this leads systematically to a N=1,S=7 cycle at a1=1)

```
   amin relfreq%     N    S   vector
-----------------------------------------------------------------------------
   125  0.800000    1    2  [2]
    -------------------------------------------------------------------------
    25  0.586666    1    3  [3]
     1  0.546666    1    7  [7]
    -   -   -   -   -   -   -   -   -   -   -   -   -   -   -   -   -   -   -
```

```
 475  1.953333    3    5  [1, 1, 3]
 575  0.353333    3    5  [1, 2, 2]
  85  7.526666    4    8  [2, 1, 2, 3]
  35  8.473333    8   16  [1, 1, 1, 1, 2, 5, 1, 4]
4675  0.160000   17   27  [1, 1, 1, 1, 1, 2, 1, 1, 2, 1, 2, 3, 2, 1, 1, 1, 5]
8675  0.146666   17   27  [1, 1, 1, 1, 3, 1, 1, 1, 1, 2, 2, 4, 1, 2, 1, 2, 2]
  47 65.320000   19   33  [1, 2, 1, 1, 1, 1, 1, 1, 1, 3, 3, 1, 1, 2, 3, 1, 2, 1, 6]
 143  8.140000   19   33  [1, 2, 1, 2, 1, 1, 3, 1, 2, 1, 1, 2, 4, 1, 2, 2, 1, 1, 4]
 899  5.993333   74  118  [1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 3, 1, 1, 2, 1, 2, 1, 1, 1, 2, 1, 2, 1, 1, 5, 4, 1, 1, 1, 1, 1, 1, 1, 5, 1,
                           1, 2, 1, 1, 2, 2, 1, 1, 1, 2, 1, 1, 2, 2, 1, 4, 1, 1, 1, 1, 1, 3, 4, 4, 1, 1, 1, 1, 1, 2, 1, 2, 1, 4,
                           2, 1, 1, 3, 2, 2]
```

r=105

```
   a_min relfreq%     N    S   vector
--------------------------------------------------------------------------------
     105 2.85000      1     2  [2]
    ----------------------------------------------------------------------------
      21 2.11000      1     3  [3]                                                        3·7·T_5( 1)  3·T_35(7)
     399 6.94000      3     5  [1, 1, 3]                                                  3·7·T_5(19)  3·T_35(133)
     483 1.30000      3     5  [1, 2, 2]                                                  3·7·T_5(23)  3·T_35(161)
    3927 0.570000    17    27  [1, 1, 1, 1, 1, 2, 1, 1, 2, 1, 2, 3, 2, 1, 1, 1, 5]        3·7·T_5(187) 3·T_35(1309)
    7287 0.510000    17    27  [1, 1, 1, 1, 3, 1, 1, 1, 1, 2, 2, 4, 1, 2, 1, 2, 2]        3·7·T_5(347) 3·T_35(2429)

      75 17.1500      2     4  [1, 3]                                                     3·5·T_7( 5)  3·T_35(25)

      39 18.4900      4     8  [1, 1, 1, 5]                                                            3·T_35(13)
      51 50.0800      4     8  [1, 2, 1, 4]                                                            3·T_35(17)
--------------------------------------------------------------------------------
```

*Note: "Zsigmondy-effect": there are **no** new cycles compared to $T_{35}()$ which were expected after r=105 is composite by 3·35*

r=139

```
  a_min relfreq%  N    S   vector
--------------------------------------------------------------------------------
   139 0.720    1    2  [2]
    ----------------------------------------------------------------------------
    11  99.3   74  136  [2, 2, 2, 1, 1, 1, 1, 2, 1, 1, 2, 2, 2, 3, 1, 4, 1, 2, 3, 1, 1, 1, 3, 1, 1, 2, 2, 2, 1, 2, 2, 1, 1, 4, 1,
                         1, 1, 3, 3, 2, 1, 2, 1, 1, 1, 1, 3, 2, 1, 2, 2, 3, 1, 1, 1, 1, 1, 1, 2, 2, 2, 2, 2, 4, 1, 2, 2, 2, 4, 4,
                         2, 1, 3, 5]
```

r=385 (=5·7·11 displays collection of all cycles of all $T_t(\ )$ where t is a divisor of r, plus some "non-explained" cycles)

```
   a_min relfreq%    N    S   vector                                                          a multiple of T_xxx()
--------------------------------------------------------------------------------
     385 0.26000     1     2  [2]                                                                   5*7*11*T_1 (  1)
    ----------------------------------------------------------------------------
      77 0.17000     1     3  [3]                                                                     7*11*T_5 (  1)
    1463 0.69000     3     5  [1, 1, 3]                                                               7*11*T_5 ( 19)
    1771 0.14000     3     5  [1, 2, 2]                                                               7*11*T_5 ( 23)
   14399 0.02000    17    27  [1, 1, 1, 1, 1, 2, 1, 1, 2, 1, 2, 3, 2, 1, 1, 1, 5]                     7*11*T_5 (187)
   26719 0.02000    17    27  [1, 1, 1, 1, 3, 1, 1, 1, 1, 2, 2, 4, 1, 2, 1, 2, 2]                     7*11*T_5 (347)

     275 1.56000     2     4  [1, 3]                                                                  5*11*T_7 (5)

      35 0.54000     2     6  [1, 5]                                                                  5* 7*T_11( 1)
     455 2.05000     8    14  [1, 1, 2, 2, 1, 1, 3, 3]                                                5* 7*T_11(13)

     143 1.77000     4     8  [1, 1, 1, 5]                                                              11*T_35(13)
     187 4.46000     4     8  [1, 2, 1, 4]                                                              11*T_35(17)

       7 2.16000     4    12  [1, 1, 2, 8]                                                               7*T_55( 1)
      49 5.65000     2     6  [2, 4]                                                                     7*T_55( 7)
     287 2.58000    16    28  [1, 1, 1, 1, 2, 1, 1, 3, 1, 2, 2, 4, 1, 2, 2, 3]                           7*T_55(41)

       5 15.5900    16    38  [4, 2, 1, 3, 2, 1, 2, 2, 1, 2, 1, 1, 2, 4, 1, 9]                           5*T_77( 1)

----------------------------------------------------------------------------------------------------------- "unexplained" (="primitive") cycles
      17 13.7200     6    18  [2, 3, 2, 1, 5, 5]                                                           T_385( 17)
      23 17.4500    20    40  [1, 1, 6, 1, 1, 1, 1, 2, 1, 2, 2, 1, 1, 2, 1, 3, 1, 1, 4, 7]                 T_385( 23)
     107 31.1700    48    84  [1, 2, 2, 1, 1, 1, 2, 1, 1, 1, 2, 2, 1, 2, 1, 1, 3, 1, 4, 1, 1, 1, 3, 3, 1, 2,       T_385(107)
                                3, 1, 3, 1, 1, 1, 1, 1, 1, 1, 2, 2, 4, 1, 1, 3, 2, 2, 2, 2, 4, 3]
```

Gottfried Helms, 3.9.2017 (prev:25.8.2017; first version 8.8.2017)