The cited statement of Dan Asimov in the thread "Bummer", translated to my T- /U-tetration conversion scheme and the powerseries in (x,u^h) , means the following

Let b=sqrt(2), $b=t^{1/t}$ $u = log(t)$

then, by the conversion formula for **T**-/**U**-tetration

(1) $T_b^{oh}(x) = (U_t^{oh}(x/t - 1)+1)*t$
$= U_t^{oh}(x')''$ *for shortness*

where I say x' is the ***shifted*** x and $U()''$ is the ***reshifted*** $U()$.

The base t for **U**-tetration is a fixpoint of base b in **T**-tetration. If two different t_1 and t_2 are inserted, both giving $t_1^{1/t1} = t_2^{1/t2} = b$ then, according to Dan Asimov it should be

(2) *if* $t_1=2$, $t_2=4$, *then if also h is non-integer then*
$(U_{t1}^{oh}(x/t_1 - 1)+1)*t_1 \neq (U_{t2}^{oh}(x/t_2 - 1)+1)*t_2$ *// D.Asimov translated*

My **U**-tetration powerseries is the following for a general t and x:

Let $U_t^{\circ h}(x)$ denote the h'th iterate of $U_t(x)$, and $u=log(t)$ then its powerseries is:

(3) $$U_t^{\circ h}(x) = a_{1,u}\frac{x}{1!} + a_{2,u}\frac{u}{u-1}\frac{x^2}{2!} + a_{3,u}\frac{u^2}{(u-1)(u^2-1)}\frac{x^3}{3!} + ... + a_{k,u}\frac{u^{k-1}}{\prod_{j=1}^{k-1}(u^j-1)}\frac{x^k}{k!} + ...$$

(for coefficients a_k see end of text)

The difference of the two different reshifted **U**-functions on shifted x should be zero if the solutions for different fixpoints are equal and should be non-zero, if D.Asimovs statement holds.

The two fixpoints in question are $t_1=2$ and $t_2=4$, let's denote

(4) $u = u_2 = log(2)$ $u_4 = log(4) = 2\ u_2$

Shifted x is $x'=x/t - 1$, so for the two fixpoints we get two different x':

(5) $x' = x_2' = x/2 - 1$ $x_4' = x/4 - 1$;

Reshifted functions $U^{oh}()$ are according to *(1)* (omitting h and x-parameter here):

(6) $U_2'' = (U_2+1)*2$ $U_4'' = (U_4+1)*4$

So we have the question of equalities in

$$(U_2^{\circ h}(\frac{x}{2}-1)+1)*2 \stackrel{?}{=} (U_4^{\circ h}(\frac{x}{4}-1)+1)*4$$

(7) $$2U_2^{\circ h}(\frac{x}{2}-1) - 4U_4^{\circ h}(\frac{x}{4}-1) - 2 \stackrel{?}{=} 0$$

$$U_2^{\circ h}(\frac{x}{2}-1) - 2U_4^{\circ h}(\frac{x}{4}-1) \stackrel{?}{=} 1$$

and, assuming the commonly understood tetration has $x=1$ this is

(7.1) $$U_2^{\circ h}(-\frac{1}{2}) - 2U_4^{\circ h}(-\frac{1}{2}-\frac{1}{4}) \stackrel{?}{=} 1$$

Read $x' = x/2 - 1$, $u = u_2$ then we get termwise, with some factoring and transfering powers of 2 according to $u_4 = 2 * u_2$

$$
\begin{aligned}
1 \overset{?}{=} & \left(a_{1,2}x' - a_{1,4}(x'-1)\right)\frac{1}{1!} \\
& + \left(\frac{a_{2,2}}{u-1}x'^2 - \frac{a_{2,4}}{2u-1}(x'-1)^2\right)\frac{u}{2!} \\
& + \left(\frac{a_{3,2}}{(u-1)(u^2-1)}x'^3 - \frac{a_{3,4}}{(2u-1)(4u^2-1)}(x'-1)^3\right)\frac{u^2}{3!} \\
& + \dots \\
& + \left(\frac{a_{k,2}x'^k}{\prod_{j=1}^{k-1}(u^j-1)} - \frac{a_{k,4}(x'-1)^k}{\prod_{j=1}^{k-1}(2^j u^j-1)}\right)\frac{u^{k-1}}{k!} \\
& + \dots
\end{aligned}
$$

where

$a_{1,2} = \quad 1\, u^h$	$a_{1,4} = \quad 1 * 2^h u^h$
$a_{2,2} = \; -(1)\, u^h$ $+(1)\, u^{2h}$	$a_{2,4} = \; -(1)\, 2^h u^h$ $+(1)\, 2^{2h} u^{2h}$
$a_{3,2} = \quad (1 + 2u)\, u^h$ $-(3 + 3u)\, u^{2h}$ $+(2 + 1u)\, u^{3h}$	$a_{3,4} = \quad (1 + 2*2u)\, 2^h\, u^h$ $-(3 + 3*2u)\, 2^{2h} u^{2h}$ $+(2 + 1*2u)\, 2^{3h} u^{3h}$
$a_{4,2} = \; -(1 + 6u + 5u^2 + 6u^3)\, u^h$ $+(7 + 18u + 18u^2 + 11u^3)\, u^{2h}$ $-(12 + 18u + 18u^2 + 6u^3)\, u^{3h}$ $+(6 + 6u + 5u^2 + 1u^3)\, u^{4h}$	$a_{4,4} = \; -(1 + 6*2u + 5*2^2u^2 + 6*2^3u^3)\, 2^h u^h$ $+(7 + 18*2u + 18*2^2u^2 + 11*2^3u^3)\, 2^{2h} u^{2h}$ $-(12 + 18*2u + 18*2^2u^2 + 6*2^3u^3)\, 2^{3h} u^{3h}$ $+(6 + 6*2u + 5*2^2u^2 + 1*2^3u^3)\, 2^{4h} u^{4h}$

Surprisingly, the terms come out to be very different, for U_4 the series is divergent and still the Euler-summed results match good (resp my poor Euler-sum-procedure for the U_4-series), see last table at end of article. One would assume an ***arbitrary*** difference, once accepted, that there is one at all – but that it is ***small*** seems then strange.

I'm speculating, whether we may find a relation to the small differences when summing **U**- and **T**-Tetra-series **AS** of increasing negative heights serially and via matrix-method: the difference is also small and seems to be even systematic (sinus-form with appropriate parameters). We are possibly touching here some more general and possibly deep number-theoretical properties concerning divergent series...

Gottfried Helms, 22.01.2008

Terms of powerseries $U_2^{\circ h}(x_2')$ and $U_4^{\circ h}(x_4')$

u_2=log(2) convergent, h=1.5, x_2'=-1/2	u_4=log(4) = 2*u_2 , h=1.5, x_4'=-3/4
0	0 (k=1)
-0.288541440693	-1.22417765620
0.0689126280100	1.04157998652
-0.0127507837732	-0.687124082870
0.00204136424674	0.384545940822
-0.000293228470888	-0.189752199617
0.0000383194808273	0.0846482973062
-0.00000462370885001	-0.0347593708687
0.000000525236845411	0.0132936456192
-0.0000000569281723993	-0.00477997168133
0.00000000581584915247	0.00162905727573
-0.000000000538064620712	-0.000528188130097
...(terms omitted up to near k=96)	...(terms omitted up to near k=96)
4.58480744730E-39	-1.59584846362
-2.75065355301E-39	-1.72389754050
1.51431659426E-39	-1.58573407414
-7.83270897747E-40	-0.940661730599
3.84801021962E-40	0.584835125611
-1.80299651796E-40	3.55058312013
8.05115179089E-41	8.77032970348
-3.40542142770E-41	17.3933076369
1.34461408871E-41	30.9964022731
-4.79544459444E-42	51.6777947254
1.41343944945E-42	82.1331378007 (k=96)

Approximations of $T_b^{\circ 1.5}(1)$ via conversion into U_2 and U_4 , last terms up to k=96

Eulersum of ($U_2^{\circ 1.5}(x_2')$+1)*2 Eulersum-order PkPow(1.0,1.4)	Eulersum of ($U_4^{\circ 1.5}(x_4')$+1)*4 Eulersum-order by PkPow(1.3,1.4)
1.53880541744	1.53880541461
1.53880541744	1.53880541629
1.53880541744	1.53880541805
1.53880541744	1.53880541983
1.53880541744	1.53880542159
1.53880541744	1.53880542326
1.53880541744	1.53880542479
1.53880541744	1.53880542613
1.53880541744	1.53880542725
1.53880541744	1.53880542810
1.53880541744	1.53880542866
1.53880541744	1.53880542891
1.53880541744	1.53880542882
1.53880541744	1.53880542841
1.53880541744	1.53880542765
1.53880541744	1.53880542657
1.53880541744	1.53880542518
1.53880541744	1.53880542350
1.53880541744	1.53880542156
1.53880541744	1.53880541939