2-step-cycles in the $mx+1$-problem

1. Basic notation and formulae

We look at a two equation-system in positive integers a,b,m,A,B , where -being a generalization for the Collatz-problem when $m=3$ - only ***positive odd*** a,b,m are considered:

(1.1) $b = (m\cdot a+1)/2^A$ $a = (m\cdot b+1)/2^B$ where $A,B = v_2(m\cdot a+1),v_2(m\cdot b+1)$ respectively

We define also (in accordance to my other articles on the Collatz-problem) the symbols $N=2$ (2 steps), and $S=A+B$.

The trivial ***product-equation*** $a\cdot b = (m\cdot a+1)/2^A\cdot (m\cdot b+1)/2^B$ leads to a formula which allows easily the determination of m from given S *(the application is under (2.1))*:

$2^S = (m +1/a)\cdot(m+1/b)$ or

(1.2) $2^S = m^2 + m(1/a+1/b) + 1/(a\cdot b)$

Using (1.1) and inserting and expanding we get a determination-formula for a and b from S and m:

$a = (m\cdot((m\cdot a+1)/2^A)+1)/2^B$
$a = (m^2a + m + 2^A)/2^{A+B}$
$a(2^S - m^2) = m + 2^A$ ===>

(1.3) $a = (m + 2^A)/(2^S - m^2)$ and accordingly
$b = (m + 2^B)/(2^S - m^2)$

2. Possible values for m depending on S

The possibility of determination of m from S comes from the fact, that formula (1.2) gives intervals of $0..1$ by intervals of $1..oo$ for a and b.

a) For the ***smallest*** numbers combination $(a,b)=(1,1)$ evaluation of the first form of (1.2) gives

(2.1) $2^S = (m+1)^2$

So we must have $m+1=2^{S/2}$ and demanding that m is integer and odd this requires

1) S must be even, say $S=2\cdot T$ and

2) m is one below 2^T: $m=2^T-1$.

Note: *if $a=b$ then of course always also $A=B$ and thus $S=2\cdot A$ must be even.*

b) For the ***largest*** numbers combination $(a,b)=(oo,oo)$ we get this as limit case

(2.2) $2^S = (m+1/oo)^2 = m^2$

and of course for all positive integer selections of $(a,b) \in N\backslash 2$ we have then

$m^2< 2^S <= (m+1)^2$

(2.3) $m < 2^{S/2} <= m+1$

This gives -for even or odd S-, that

(2.4) $m = floor(2^{S/2})$.

Note: *we don't look at $m=ceil(2^{S/2})$ at the moment because this would lead to discussion of negative values in (a,b)*

The above derivation shows,

- first, that we need not consider ***all*** natural numbers as candidates for m, but that we can use the values S and derive the possible values m.

- second, that we need only look at odd S after we know, that for even S we get $m=2^{S/2}-1$ and for that values we'll get the only solutions $a=b=1$, the trivial cycle.

- There is a third reductive rule: we want only consider odd m. This leaves us finally with a set of pairs

(2.5) $(S,m) \in \{ (1,1), (5,5), (7,11), (11,45), (15,181), (27,11585), (33,92681), (35,185363),... \}$

3. Checking for 2-step-cycles in the reduced set of possible cases

The first approach -for small numbers S- is now, to insert acceptable values for the pair (S,m) and check numerically the possible existence of 2-step-cycles.

"Exponents-method": We can test along the exponents $A<B$ (of course with $A+B=S$ or $B=S-A$). For this we can use the determination-formula for the elements a (and b)

[from (1.3):]

(3.1) $a = (m+2^A)/(2^S - m^2)$ $b=(m+2^{S-A})/(2^S - m^2)$

and for each S insert successively integer values values for A in $1...S/2$. For instance for $S=35$, $A=1..17$ this needs $t_{35}=17$ numerical tests and can thus detect the small-number-solutions $(S,m)=(5,5)$, $(a,b)=(1,3)$ and $(S,m)=(15,181)$, $(a,b)=(27,611)$ and $(a,b)=(35,99)$ and find that up to $m=1.8 E5$ that there are no more solutions using only $t_{all} =1+2+3+5+7+13+16+17 = 64$ numerical tests.

"Mean-method": A second option is to use the product formula $2^S = (m+1/a)(m+1/b)$ and follow the searchspace for the (smaller) element $a<b$ which has an upper bound a_m by $2^S = (m+1/a_m)(m+1/a_m) = (m+1/a_m)^2$

(3.2) $1<= a< a_m = 1/(2^{S/2}-m)$

This gives for the examples $S<=35$ the list

Table (3.3)

S	m	a_m	a	t_S=# of tests
1	1	2.41421356237	1	1
5	5	1.52240774993	1	1
7	11	3.18767264271	1,3	2
11	45	3.92412321721	1,3	2
15	181	51.7170479977	1,3,5,...,51	25
27	11585	4.21047383299	1,3	2
33	92681	1.11108187341	1	1
35	185363	1.24992599452	1	1
				sum: $t_{all}=35$

and this needs $t_{all}=35$ tests which is less than with the previous method.

"Combined": Combining that two methods, first computing the upperbound a_m, and if a_m is small, then apply the "mean-method" and if $a_m > S$ then apply the "exponent-method". We need then only $t_{all}=17$ tests to arrive at our result for S up to 35. (If we look at S up to $S=299$, we need $t_{all}=164$ tests to disprove any 2-step-cycle for $181<m<1e45$ and if we look up to $S=2995$ we need $t_{all}=2951$ tests for this disproof up to $m<6.2E450$) .

Unfortunately, for this problem-configuration I don't see any possibility to apply something like the 1-cycle-disproof for the $3x+1$-problem according to the Steiner/Simons - method with the Rhin-bound for the distance $S\cdot log(2) - N\cdot log(3)$. The Rhin-bound depends on parameter N and supplies some upperbound for N, while we have here a fixed value for $N=2$ in the case before us.

4. How to handle the lack of an analytical bound for S or m to small numbers?

Combining the two steps in (1.1) we can derive equations for a and b as given above from (1.3):

(4.1) $a = (m+2^A)/(2^S - m^2)$ $b=(m+2^{S-A})/(2^S - m^2)$

In this equations we find, that if there is a 2-step-cycle then a and b are integer, and the terms $m+2^A$ as well as $m+2^B$ must be divisible by the denominator 2^S-m^2. I didn't succeed yet to find an analytical argument that allows to deduce the impossibility of such integrality by the parameter S (and/or m) alone. Fixing $a=1$ allows however a disproof for 2-step-cycles other then for $(S,m)=(5,5)$, but no success for keeping a indetermined. A proof for the nonexistence of solutions $S>5$ has been developed in an answer to a question of mine in the forum MathOverflow.

"Cyclic group-method": Towards the generalization to **both** a and b to be left indeterminate, a better approach seems to me now is to look at the difference $b-a$, which cancels the depending variable m in the numerator:

(4.1) $b-a = (2^{S-A} - 2^A)/(2^S - m^2) = 2^A\cdot(2^{S-2A} - 1)/(2^S - m^2)$

Let's denote the overall important denominator's structure: $D=2^S - m^2$. Then, because the numerator must be divisible by D, we have the property that the exponent $(S-2A)$ in the numerator's parenthese must equal the multiplicative order of D to base 2, or equal a multiple of it:

$o = order_2(D)$ // = multiplicative order D to base 2 with $D|2^o - 1$
$S - 2A = k\cdot o$
(4.2) ==> $o <= S - 2$

Here we see, that the upper bound for o is $S-2$ to make a 2-step-cycle possible at all. But looking empirically at that values for o we find an **exponential** relation between o and S, with the only four examples of $o<=S-2$ and 2-step-cycles only for $(S,m)=(5,5)$ and for $(S,m)=(15,181)$ (where the latter configuration allows even **two** 2-step-cycles). See two pictures, plotting empirical data up to $S=299$ (data table at the end of the paper).

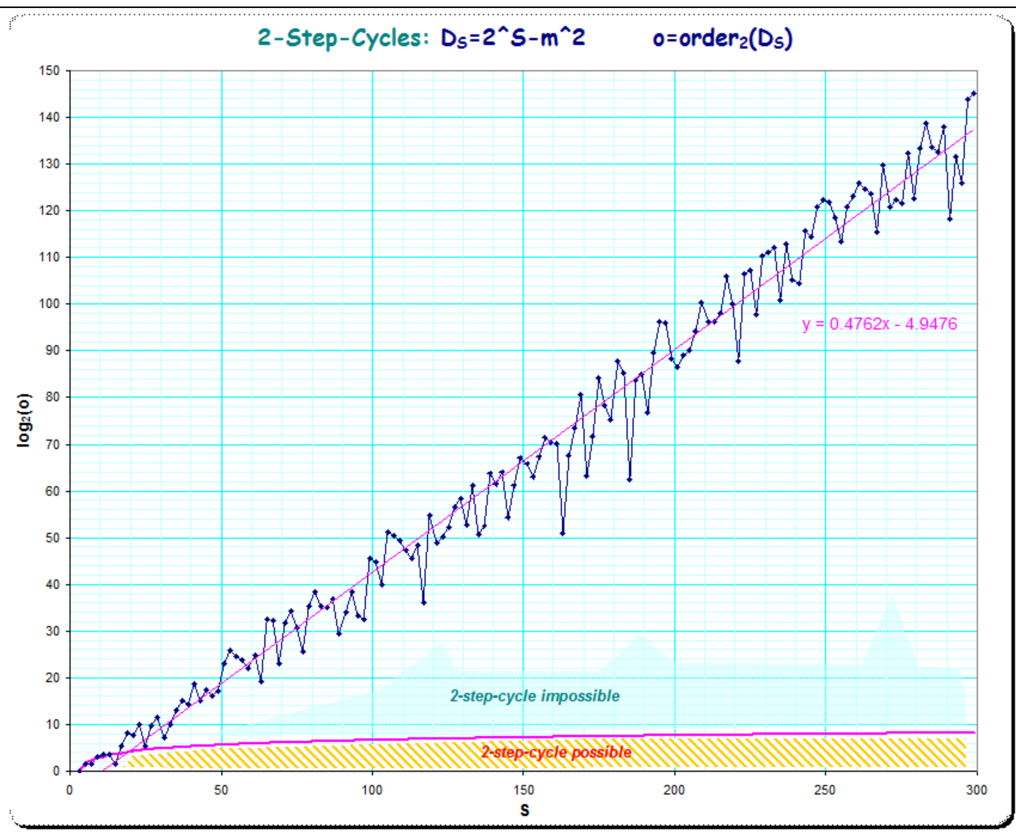
(S,m), $D=2^S-m^2$, $o=$ znorder(Mod(2,D)) [Pari/GP code]

Question: is $o<=S-2$?

1) Overview (S,m) for $S=3$ to 299.

We see a clear linear tendency in the logarithmic scale, such that o grows **exponentially over** S, and is much larger than S already from small S on. The extremality of the locally lowest values of o very suggestively seem to not to account for the exponential growth of o.

Trend of $o(S)$ is about $2^{S\cdot 0.476}$:

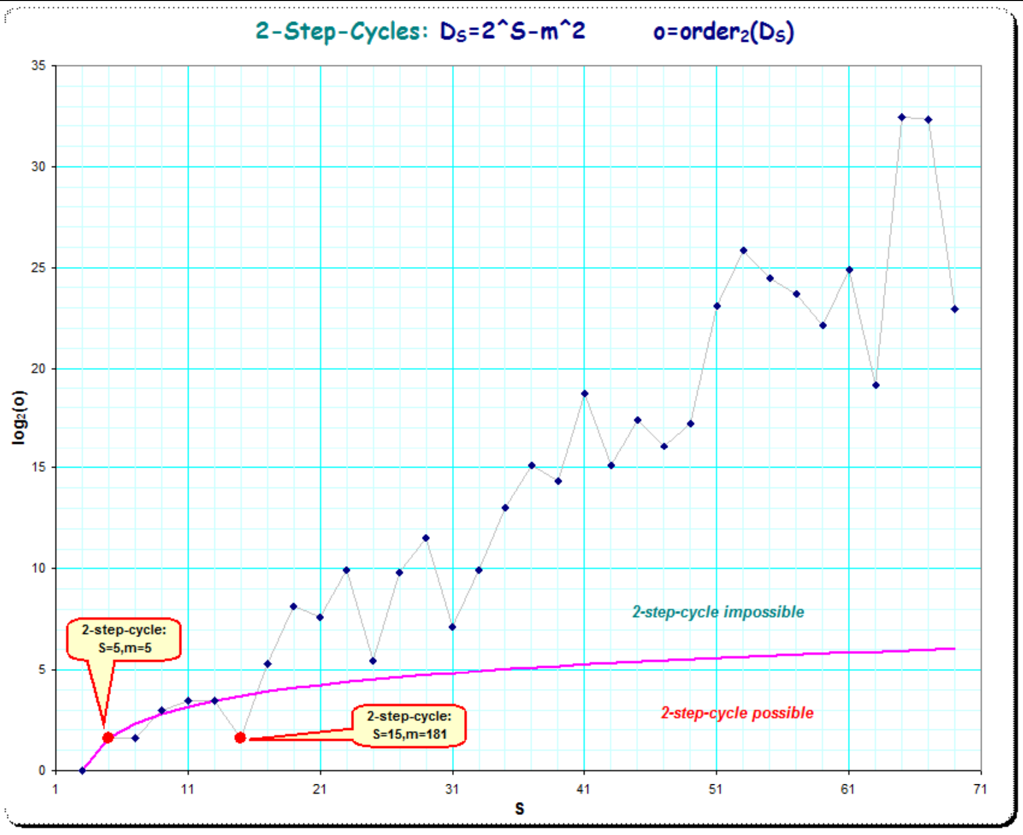


2) Detail for small *S*

We see (blue vs pink line) that the only cases where $o<=S-2$ are that at

S= 5:	*(S,m)=(5,5)) o=3 ,*
S= 7:	*(S,m)=(7,11) o=3,*
S=13:	*(S,m)=(13,91) o=11,*
S=15:	*(S,m)=(15,181) o=3*

where only *(S,m)=(5,5)* and *(S,m)=(15,181)* have one or two 2-step-cycles.



Unfortunately, no approximation bound for 2^S to m^2 is known to me; for getting some insight I transformed that problem to one of the bit-patterns of the integer and fractional parts of $2^T \cdot sqrt(2)$, but again I've been missing any usable upper or lower bounds in terms of *T* so far.

Appendix

5. Table 1: Table of data for *S=3..299* (Excel-notation)

Note: we look at multiplicative order of $|D|$ in relation to S for the imagination of some trend. Thus we show the values for $m=floor(2^{S/2})$ ***and*** $m=ceil(2^{S/2})$ - always the next odd value of m near $2^{S/2}$ - all for odd S only. "$Ub(o)$" means: upper bound for o by $o<=S-2$.

S	m	|D|	o	Ub(o) =S-2	log2(o)	log2(Ub(o))	S	m	|D|	o	Ub(o) =S-2	log2(o)	log2(Ub(o))
							151	5.34E+22	9.04E+22	5.88E+19	149	65.67156	7.21916852
3	3	1	1	1	0	0	153	1.07E+23	1.48E+23	8.60E+18	151	62.8983897	7.23840474
5	5	7	3	3	1.5849625	1.5849625	155	2.14E+23	1.64E+23	1.98E+20	153	67.4204787	7.25738784
7	11	7	3	5	1.5849625	2.32192809	157	4.27E+23	1.98E+23	2.90E+21	155	71.2985509	7.27612441
9	23	17	8	7	3	2.80735492	159	8.55E+23	9.20E+23	1.53E+21	157	70.3700975	7.29462075
11	45	23	11	9	3.45943162	3.169925	161	1.71E+24	2.59E+23	1.35E+21	159	70.193172	7.31288296
13	91	89	11	11	3.45943162	3.45943162	163	3.42E+24	5.80E+24	2.28E+15	161	51.0148012	7.33091688
15	181	7	3	13	1.5849625	3.70043972	165	6.84E+24	9.53E+24	2.36E+20	163	67.6785222	7.34872815
17	363	697	40	15	5.32192809	3.9068906	167	1.37E+25	1.08E+25	1.20E+22	165	73.349075	7.36632221
19	725	1337	285	17	8.15481811	4.08746284	169	2.74E+25	1.16E+25	1.94E+24	167	80.6814278	7.38370429
21	1449	2449	195	19	7.60733031	4.24792751	171	5.47E+25	6.29E+25	1.12E+19	169	63.2776655	7.40087944
23	2897	4001	1000	21	9.96578428	4.39231742	173	1.09E+26	3.27E+25	3.53E+21	171	71.5785317	7.41785251
25	5793	4417	45	23	5.4918531	4.52356196	175	2.19E+26	3.07E+26	2.41E+25	173	84.3150785	7.43462823
27	11585	5503	917	25	9.84077792	4.64385619	177	4.38E+26	3.52E+26	3.88E+23	175	78.3616715	7.45121111
29	23171	24329	3041	27	11.5703301	4.7548875	179	8.75E+26	3.43E+26	4.38E+22	177	75.2126266	7.46760555
31	46341	4633	140	29	7.12928302	4.857981	181	1.75E+27	2.13E+27	2.66E+26	179	87.7822997	7.48381578
33	92681	166831	993	31	9.95564991	4.95419631	183	3.50E+27	1.51E+27	4.73E+25	181	85.2898801	7.49984589
35	185363	296599	8568	33	13.0647428	5.04439412	185	7.00E+27	7.95E+27	6.78E+18	183	62.5554914	7.51569984
37	370727	444943	35805	35	15.1278734	5.12928302	187	1.40E+28	3.79E+27	1.51E+25	185	83.6386276	7.53138146
39	741455	296863	21204	37	14.3720488	5.20945337	189	2.80E+28	4.08E+28	3.52E+25	187	84.8618925	7.54689446
41	1482911	1778369	438945	39	18.7436807	5.28540222	191	5.60E+28	5.13E+28	1.33E+23	189	76.8137112	7.56224242
43	2965821	1181833	36860	41	15.1697685	5.357552	193	1.12E+29	1.87E+28	9.35E+26	191	89.5953417	7.57742883
45	5931641	7135951	177460	43	17.4371343	5.42626475	195	2.24E+29	3.73E+29	9.33E+28	193	96.2358463	7.59245704
47	11863283	4817239	69384	45	16.0823154	5.4918531	197	4.48E+29	5.96E+29	7.46E+28	195	95.9122376	7.60733031
49	23726567	28184177	155265	47	17.2443731	5.55458885	199	8.96E+29	5.93E+29	3.82E+26	197	88.3019249	7.62205182
51	47453133	17830441	8915220	49	23.087839	5.61470984	201	1.79E+30	1.21E+30	1.09E+26	199	86.4905242	7.63662462
53	94906265	118490767	59245383	51	25.8201994	5.67242534	203	3.59E+30	2.32E+30	6.39E+26	201	89.0459171	7.65105169
55	189812531	94338007	23009260	53	24.4557113	5.72792045	205	7.17E+30	5.08E+30	1.26E+27	203	90.0237879	7.66533592
57	379625063	381898097	13601136	55	23.6972238	5.78135971	207	1.43E+31	8.37E+30	2.18E+28	205	94.1389352	7.6794801
59	759250125	9092137	4546068	57	22.1161878	5.83289001	209	2.87E+31	2.39E+31	1.49E+30	207	100.234852	7.69348696
61	1518500249	3000631951	30368188	59	24.8560575	5.88264305	211	5.74E+31	1.93E+31	8.73E+28	209	96.1396275	7.70735913
63	3037000499	5928526807	587832	61	19.1650444	5.93073734	213	1.15E+32	1.52E+32	9.78E+28	211	96.3040769	7.72109919
65	6074000999	1.1566E+10	5783052615	63	32.4291841	5.97727992	215	2.29E+32	1.51E+32	2.99E+29	213	97.9141299	7.73470962
67	1.2148E+10	2.1968E+10	5358150460	65	32.3190879	6.02236781	217	4.59E+32	3.15E+32	7.41E+31	215	105.869141	7.74819285
69	2.4296E+10	3.9282E+10	8166120	67	22.9612193	6.06608919	219	9.18E+32	5.76E+32	1.41E+30	217	100.150137	7.76155123
71	4.8592E+10	5.9943E+10	3745488406	69	31.8025067	6.10852446	221	1.84E+33	1.37E+33	2.80E+26	219	87.8575484	7.77478706
73	9.7184E+10	4.5402E+10	2.2701E+10	71	34.4020514	6.14974712	223	3.67E+33	1.88E+33	1.17E+32	221	106.530741	7.78790256
75	1.9437E+11	2.0713E+11	1849236765	73	30.7842828	6.18982456	225	7.34E+33	7.18E+33	1.98E+32	223	107.287257	7.8008999
77	3.8874E+11	5.1033E+10	53158644	75	25.663801	6.22881869	227	1.47E+34	6.33E+32	2.69E+29	225	97.7633806	7.81378119
79	7.7747E+11	1.35E+12	3.9716E+10	77	35.2090026	6.26678654	229	2.94E+34	5.62E+34	1.75E+33	227	110.42781	7.82654849
81	1.55E+12	2.29E+12	3.8223E+11	79	38.47564	6.30378075	231	5.87E+34	1.07E+35	2.67E+33	229	111.040522	7.83920379
83	3.11E+12	2.95E+12	4.5574E+10	81	35.4074823	6.33985	233	1.17E+35	1.94E+35	6.07E+33	231	112.224827	7.85174904
85	6.22E+12	6.2484E+11	3.9047E+10	83	35.1845085	6.37503943	235	2.35E+35	3.08E+35	2.23E+30	233	100.813595	7.86418614
87	1.24E+13	2.24E+13	1.2891E+11	85	36.9075325	6.40939094	237	4.70E+35	2.92E+35	8.91E+33	235	112.778302	7.87651695
89	2.49E+13	3.98E+13	774666090	87	29.5289993	6.4429435	239	9.40E+35	7.11E+35	4.36E+31	237	105.103736	7.88874325
91	4.98E+13	5.95E+13	1.9261E+10	89	34.164973	6.47573343	241	1.88E+36	9.14E+35	2.81E+31	239	104.469413	7.90086681
93	9.95E+13	3.91E+13	3.6405E+11	91	38.4053412	6.50779464	243	3.76E+36	3.86E+36	6.90E+34	241	115.731887	7.91288934
95	1.99E+14	2.42E+14	9807566832	93	33.1912481	6.53915881	245	7.52E+36	4.16E+35	2.60E+34	243	114.323781	7.9248125
97	3.98E+14	1.71E+14	5783848224	95	32.4293825	6.56985561	247	1.50E+37	2.84E+37	2.37E+36	245	120.832945	7.93663794
99	7.96E+14	9.09E+14	5.67E+13	97	45.6876249	6.59991284	249	3.01E+37	5.35E+37	6.69E+36	247	122.33086	7.94836723
101	1.59E+15	4.50E+14	3.21E+13	99	44.868007	6.62935662	251	6.02E+37	9.37E+37	5.20E+36	249	121.969298	7.96000193
103	3.18E+15	4.57E+15	9.7172E+11	101	39.821753	6.65821148	253	1.20E+38	1.34E+38	4.89E+35	251	118.557265	7.97154355
105	6.37E+15	5.55E+15	2.77E+15	103	51.3003344	6.68650053	255	2.41E+38	5.53E+37	1.29E+34	253	113.314636	7.98299357
107	1.27E+16	3.29E+15	1.65E+15	105	50.5482097	6.71424552	257	4.81E+38	7.41E+38	2.32E+36	255	120.801073	7.99435344
109	2.55E+16	3.78E+16	7.87E+14	107	49.4835132	6.74146699	259	9.62E+38	1.04E+39	1.08E+37	257	123.026275	8.00562455
111	5.10E+16	4.92E+16	1.81E+14	109	47.3610221	6.76818432	261	1.92E+39	3.09E+38	7.71E+37	259	125.85892	8.01680829
113	1.02E+17	6.93E+15	5.35E+13	111	45.6054995	6.79441587	263	3.85E+39	6.47E+39	3.37E+37	261	124.662969	8.027906
115	2.04E+17	3.80E+17	3.77E+14	113	48.4211077	6.82017896	265	7.70E+39	1.05E+40	1.56E+37	263	123.549963	8.03891899
117	4.08E+17	7.04E+17	6.8624E+10	115	35.9979911	6.84549005	267	1.54E+40	1.10E+40	5.52E+34	265	115.410701	8.04984855
119	8.15E+17	1.19E+18	2.97E+16	117	54.7203041	6.87036472	269	3.08E+40	1.74E+40	1.09E+39	267	129.676042	8.06069593
121	1.63E+18	1.49E+18	4.84E+14	119	48.7830875	6.89481776	271	6.16E+40	5.36E+40	2.41E+36	269	120.857926	8.07146236
123	3.26E+18	5.70E+17	1.30E+15	121	50.2066733	6.91886324	273	1.23E+41	3.20E+40	6.94E+36	271	122.383644	8.08214904
125	6.52E+18	1.08E+19	5.40E+15	123	52.2622149	6.94251451	275	2.46E+41	3.65E+41	4.22E+36	273	121.667128	8.09275714
127	1.30E+19	1.70E+19	1.14E+17	125	56.6614078	6.96578428	277	4.93E+41	4.74E+41	6.96E+39	275	132.355319	8.10328781
129	2.61E+19	1.57E+19	3.57E+17	127	58.307715	6.98868469	279	9.86E+41	7.66E+40	8.68E+36	277	122.706818	8.11374217
131	5.22E+19	4.16E+19	6.86E+15	129	52.6074934	7.01122726	281	1.97E+42	3.64E+42	1.44E+40	279	133.40592	8.12412131
133	1.04E+20	4.25E+19	2.50E+18	131	61.11511	7.033423	283	3.94E+42	6.66E+42	5.55E+41	281	138.670808	8.13442632
135	2.09E+20	2.48E+20	1.75E+15	133	50.6391878	7.05528244	285	7.88E+42	1.09E+43	1.62E+40	283	133.56899	8.14465824
137	4.17E+20	1.56E+20	5.89E+15	135	52.3867365	7.0768156	287	1.58E+43	1.19E+43	8.47E+39	285	132.637993	8.15481811
139	8.35E+20	1.05E+21	1.45E+19	137	63.6558699	7.09803208	289	3.15E+43	1.54E+43	3.21E+41	287	137.881083	8.16490693
141	1.67E+21	8.51E+20	3.10E+18	139	61.4272001	7.11894107	291	6.31E+43	6.45E+43	4.15E+35	289	118.319856	8.17492568
143	3.34E+21	3.27E+21	1.82E+19	141	63.9769657	7.13955135	293	1.26E+44	5.75E+42	4.25E+39	291	131.643499	8.18487534
145	6.68E+21	2.57E+20	2.09E+16	143	54.2148557	7.15987134	295	2.52E+44	4.82E+44	7.69E+37	293	125.854604	8.19475685
147	1.34E+22	2.57E+22	2.48E+18	145	61.1062205	7.17990909	297	5.05E+44	9.17E+44	2.16E+43	295	143.95274	8.20457114
149	2.67E+22	4.93E+22	1.66E+20	147	67.1672565	7.19967234	299	1.01E+45	1.65E+45	5.16E+43	297	145.209883	8.21431912

6. Cycles in generalizations of the type *m·x+1* /heuristic by author 10'2020

```
forstep(j= 3,19999,2,tmp=checkloop(j,1000,30,+1);if(tmp==[],next());print(j,"  ",tmp))
forstep(j= 3, 9999,2,tmp=checkloop(j,1000,30,-1);if(tmp==[],next());print(j,"  ",tmp))
```

Cycles in Collatz-type iterations for positive and for negative numbers a_k ,
$a_{k+1} = (m \cdot a_k + 1)/2^A$ cycle of length N by $a_{N+1} = a_1$
Tested: bases m<20000 a_1<1000 length N<30 (G.Helms, 5'2016)

Tested: S 2..1999 : bases m<=7.5E300 for N=2 ... m<= 1.14E20 for N=30; main criterion: $1 \le a_1 < a_m$ (G.Helms, 10'2020)

base m		positive integers	negative integers		*comments*
m = 2^k-1	3	[1,1,...]	[-1, -1,...]		
			[-5, -7,\ -5, ...]	N=2 1-cycle	
			[-17,-25,-37,-55,\-41,-61,-91,\-17,...]	N=7 2-cycle	
	7	[1,1,...]			2^3 = (7+1)
	15	[1,1,...]			2^4 = (15+1)
	31	[1,1,...]			2^5 = (31+1)
	63	[1,1,...]			2^6 = (63+1)
	...	...			...
	16383	[1,1,...]			2^{15} = (16383+1)
	...	...			...
m = 2^k+1	3	[1,1,...] (see above)	[-1, -1,...] (see above)		2^2 = (3+1) 2^1 = (3-1)
			[-5, -7,\ -5, ...]	N=2 1-cycle	2^3 = (3-1/5)(3-1/7)
			[-17,-25,-37,-55,\-41,-61,-91,\-17,...] (see above)	N=7 2-cycle	2^{12} = (3-1/17)(3-1/25)...(3-1/91)
	5		[-1,-1,...]		
		[1, 3, \ 1, ...]		N=2 1-cycle	2^5 = (5+1)(5+1/3)
		[13, 33, 83, \ 13, ...]		N=3 1-cycle	2^7 = (5+1/13)(5+1/33)(5+1/83)
		[17, 43, 27,\ 17, ...]		N=3 2-cycle	2^7 = (5+1/17)(5+1/43)(5+1/27)
	9		[-1,-1,...]		2^3 = (9-1)
	17		[-1,-1,...]		2^4 = (17-1)
	33		[-1,-1,...]		2^5 = (33-1)
	...		...		...
	8193		[-1,-1,...]		2^{13} = (8193-1)
	...				...
other m	181	[27, 611,\ 27, ...]		N=2 1-cycle	2^{15} = (181+1/27)(181+1/611)
		[35, 99,\ 35, ...]		N=2 1-cycle	2^{15} = (181+1/35)(181+1/99)
? >20000					

The cycles m=5 : [13,33,83] and m=181 : [27,611] are also mentioned by [Crandall 1978]
Note that for m=181 we have that $m^2 = 181^2 = 32761$ is very near $2^{15} = 32768$